NAME ________________ DATE ________ CLASS ________

Refraction

Problem A

SNELL'S LAW

PROBLEM

The smallest brilliant-cut diamond has a mass of about 15 μg and a height of just 0.11 mm. Suppose a ray of light enters the diamond from the air and, upon contact with one of the gem's facets, refracts at an angle of 22.2°. If the angle of incidence is 65.0°, what is the diamond's index of refraction?

SOLUTION

Given: $\theta_i = 65.0°$ $\theta_r = 22.2°$ $n_i = 1.00$

Unknown: $n_r = ?$

Use the equation for Snell's law.

$$n_i \sin \theta_i = n_r \sin \theta_r$$

$$n_r = n_i \frac{\sin \theta_i}{\sin \theta_r} = (1.00)\frac{(\sin 65.0°)}{(\sin 22.2°)} = \boxed{2.40}$$

ADDITIONAL PRACTICE

1. Extra dense flint glass has one of the highest indices of refraction of any type of glass. Suppose a beam of light passes from air into a block of extra dense flint glass. If the light has an angle of incidence of 72° and an angle of refraction of 34°, what is the index of refraction of the glass?

2. The index of refraction of a clear oil is determined by passing a beam of light through the oil and measuring the angles of incidence and refraction. If the light in air approaches the oil's surface at an angle of 47.9° to the normal and moves into the oil at an angle of 29.0° to the normal, what is the oil's index of refraction? Assume the index of refraction for air is 1.00.

3. Someone on a glass-bottom boat shines a light through the glass into the water below. A scuba diver beneath the boat sees the light at an angle of 17° with respect to the normal. If the glass's index of refraction is 1.5 and the water's index of refraction is 1.33, what is the angle of incidence with which the light passes from the glass into the water? What is the angle of incidence with which the light passes from the air into the glass?

4. A beam of light is passed through a layer of ice into a fresh-water lake below. The angle of incidence for the light in the ice is 55.0°, while the angle of refraction for the light in the water is 53.8°. Calculate the index of refraction of the ice, using 1.33 as the index of refraction of fresh water.

5. An arrangement of three glass blocks with indices of refraction of 1.5, 1.6, and 1.7 are sandwiched together. A beam of light enters the first block from air at an angle of 48° with respect to the normal. What is the angle of refraction after the light enters the third block?

Light and Reflection

Problem B

IMAGING WITH CONCAVE MIRRORS

PROBLEM

Lord Rosse, who lived in Ireland in the nineteenth century, built a reflecting telescope called the Leviathan. Lord Rosse used it for astronomical observations and discovered the spiral form of galaxies. Suppose the Leviathan's mirror has a focal length of 2.50 m. Where would you place an object in front of the mirror in order to form an image at a distance of 3.75 m? What would the magnification be? If the image height were 6.0 cm, what would the object height be?

SOLUTION

1. DEFINE

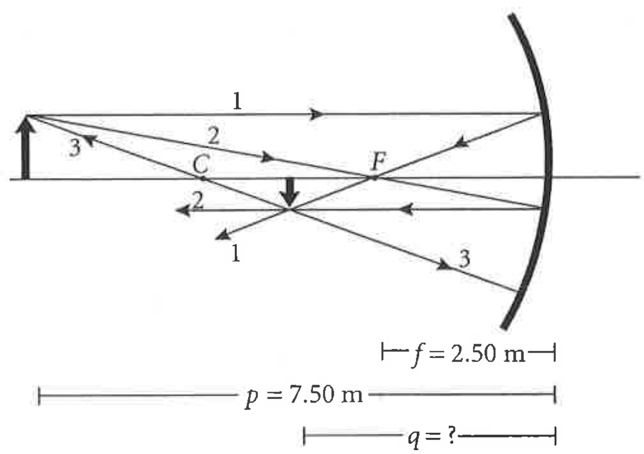
Given: $f = +2.50$ m $\quad q = +3.75$ m

$h = 6.0$ cm

The mirror is concave, so f is positive. The object is in front of the mirror, so q is positive.

Unknown: $p = ?$ $\quad M = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: Use the mirror equation for focal length and the magnification formula.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad M = -\frac{q}{p}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\frac{1}{p} = \frac{1}{2.50 \text{ m}} - \frac{1}{3.75 \text{ m}} = \frac{0.400}{1 \text{ m}} - \frac{0.267}{1 \text{ m}} = \frac{0.133}{1 \text{ m}}$$

$$p = \boxed{7.50 \text{ m}}$$

4. EVALUATE Substitute the values for p and q to find the magnification of the image and h' to find the object height.

$$M = -\frac{3.75 \text{ m}}{7.50 \text{ m}} = \boxed{-0.500}$$

$$h = -\frac{ph'}{q} = -\frac{(7.50 \text{ m})(0.060 \text{ m})}{3.75 \text{ m}} = \boxed{0.12 \text{ m}}$$

The image appears between the focal point (2.50 m) and the center of curvature, is smaller than the object, and is inverted ($-1 < M < 0$). These results are confirmed by the ray diagram. The image is therefore real.

ADDITIONAL PRACTICE

1. In Alaska, the top of Mount McKinley has been seen from the top of Mount Sanford, a distance of 370 km. An object is 3.70×10^2 km from a giant concave mirror. If the focal length of the mirror is 2.50×10^2 km what are the object distance and the magnification?

2. A human hair is about 80.0 μm thick. If one uses a concave mirror with a focal length of 2.50 cm and obtains an image of −59.0 cm, how far has the hair been placed from the mirror? What is the magnification of the hair?

3. A mature blue whale may have a length of 28.0 m. How far from a concave mirror with a focal length of 30.0 m must a 7.00-m-long baby blue whale be placed to get a real image the size of a mature blue whale?

4. In 1950 in Seattle, Washington, there was a Christmas tree 67.4 m tall. How far from a concave mirror having a radius of curvature equal to 12.0 m must a person 1.69 m tall stand to form a virtual image equal to the height of the tree? Will the image be upright or inverted?

5. A stalagmite that is 32 m tall can be found in a cave in Slovakia. If a concave mirror with a focal length of 120 m is placed 180 m from this stalagmite, how far from the mirror will the image form? What is the size of the image? Is it upright or inverted? real or virtual?

6. The eye of the Atlantic giant squid has a diameter of 5.00×10^2 mm. If the eye is viewed in a concave mirror with a radius of curvature equal to the diameter of the eye and the eye is 1.000×10^3 mm from the mirror, how far is the image from the mirror? What is the size of the image? Is the image real or virtual?

7. *Quick Bird* is the first commercial satellite designed for forming high-resolution images of objects on Earth. Suppose the satellite is 1.00×10^2 km above the ground and uses a concave mirror to form a primary image of a 1.00 m object. If the image size is 4.00 μm and the image is inverted, what is the mirror's radius of curvature?

8. A stalactite with a length of 10.0 m was found in Brazil. If the stalactite is placed 18.0 m in front of a concave mirror, a real image 24.0 m tall is formed. Calculate the mirror's radius of curvature.

Light and Reflection

Problem C

CONVEX MIRRORS

PROBLEM

The largest jellyfish ever caught had tentacles up to 36 m long, which is greater than the length of a blue whale. Suppose the jellyfish is located in front of a convex spherical mirror 36.0 m away. If the mirror has a focal length of 12.0 m, how far from the mirror is the image? What is the image height of the jellyfish?

SOLUTION

1. DEFINE

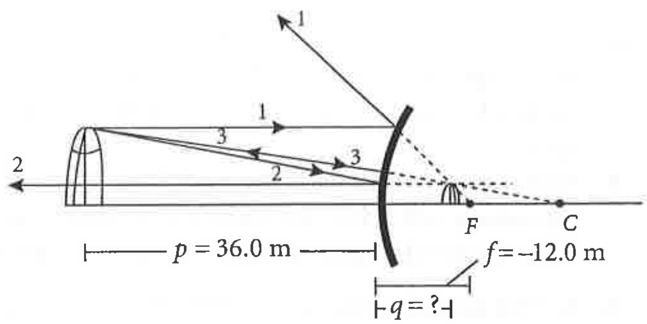
Given: $f = -12.0$ m $\quad p = +36.0$ m

$h = 36$ m

The mirror is convex, so f is negative. The object is in front of the mirror, so p is positive.

Unknown: $q = ?\quad M = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: Use the mirror equation for focal length and the magnification formula.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad M = -\frac{q}{p}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\frac{1}{q} = -\frac{1}{12.0\text{ m}} - \frac{1}{36.0\text{ m}} = -\frac{0.0833}{1\text{ m}} - \frac{0.0278}{1\text{ m}} - \frac{0.1111}{1\text{ m}}$$

$$q = \boxed{-9.001\text{ m}}$$

Substitute the values for p and q to find the magnification of the image and h to find the image height.

$$M = -\frac{-9.001\text{ m}}{36.0\text{ m}} = \boxed{0.250}$$

$$h' = -\frac{qh}{p} = -\frac{(-9.001\text{ m})(36\text{ m})}{(36.0\text{ m})} = \boxed{9.001\text{ m}}$$

The image appears between the focal point (–12.0 m) and the mirror's surface, as confirmed by the ray diagram. The image is smaller than the object ($M < 1$) and is upright ($M > 0$), as is also confirmed by the ray diagram.

ADDITIONAL PRACTICE

1. The radius of Earth is 6.40×10^3 km. The moon is about 3.84×10^5 km away from Earth and has a diameter of 3475 km. The Pacific Ocean surface, which can be considered a convex mirror, forms a virtual image of the moon. What is the diameter of that image?

2. A 10 g thread of wool was produced by Julitha Barber of Australia in 1989. Its length was 553 m. Suppose Barber is standing a distance equal to the thread's length from a convex mirror. If the mirror's radius of curvature is 1.20×10^2 m, what will the magnification of the image be?

3. Among the many discoveries made with the Hubble Space Telescope are four new moons of Saturn, the largest being just about 70.0 km in diameter. Suppose this moon is covered by a highly reflective coating, thus forming a spherical convex mirror. Another moon happens to pass by at a distance of 1.00×10^2 km. What is the image distance?

4. The largest scale model of the solar system was built in Peoria, Illinois. In this model the sun has a diameter of 11.0 m. The real diameter of the sun is 1.4×10^6 km. What is the scale to which the sun's size has been reduced in the model? If the model's sun is a reflecting sphere, where in front of the sphere is the object located?

5. Bob Henderson of Canada built a model railway to a scale of 1:1400. How far from a convex mirror with a focal length of 20.0 mm should a full-size engine be placed so that the size of its virtual image is the same as that of the model engine?

6. The largest starfish ever discovered had a diameter of 1.38 m. Suppose an object of this size is placed 6.00 m in front of a convex mirror. If the image formed is just 0.900 cm in diameter (the size of the smallest starfish), what is the radius of curvature of the mirror?

7. In 1995, a functioning replica of the 1936 Toyota Model AA sedan was made in Japan. The model is a mere 4.78 mm in length. Suppose an object measuring 12.8 cm is placed in front of a convex mirror with a focal length of 64.0 cm. If the size of the image is the same as the size of the model car, how far is the image from the mirror's surface?

8. Some New Guinea butterflies have a wingspan of about 2.80×10^2 mm. However, some butterflies which inhabit the Canary Islands have a wingspan of only 2.00 mm. Suppose a butterfly from New Guinea is placed in front a convex mirror. The image produced is the size of a butterfly from the Canary Islands. If the image is 50.0 cm from the mirror's surface, what is the focal length of the mirror?

Refraction

Problem C

CRITICAL ANGLE

PROBLEM

Rutile, TiO_2, has one of the highest indices of refraction: 2.80. Suppose the critical angle between rutile and an unknown liquid is 33.6°. What is the liquid's index of refraction?

SOLUTION

Given: $\theta_c = 33.6°$

$n_i = 2.80$

Unknown: $n_r = ?$

Use the equation for critical angle.

$$\sin \theta_c = \frac{n_r}{n_i}$$

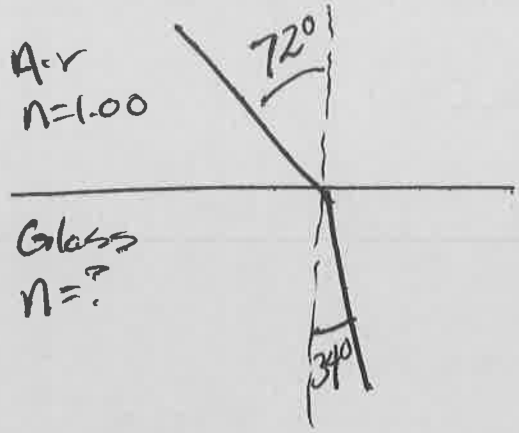
$$n_r = n_i \sin \theta_c = (2.80)(\sin 33.6°) = \boxed{1.54}$$

ADDITIONAL PRACTICE

1. Light moves from glass into a substance of unknown refraction index. If the critical angle for the glass is 46° and the index of refraction for the glass is 1.5, what is the index of refraction of the other substance?
2. The largest uncut diamond had a mass of more than 600 g. Eventually, the diamond was cut into several pieces. Suppose one of those pieces is a cube with sides 1.00 cm wide. If a beam of light were to pass from air into the diamond with an angle of incidence equal to 75.0°, the angle of refraction would be 23.3°. From this information, calculate the index of refraction and the critical angle for diamond in air.
3. A British company makes optical fibers that are 13.6 km in length. If the critical angle for the fibers in air is 42.1°, what is the index of refraction of the fiber material?
4. In 1996, the Fiberoptic Link Around the Globe (FLAG) was started. It initially involves placing a 27 000 km fiber optic cable at the bottom of the Mediterranean Sea and the Indian Ocean. Suppose the index of refraction of this fiber is 1.56 and the index of refraction of sea water 1.36, what is critical angle for internal reflection in the fiber?
5. The world's thinnest glass is 0.025 mm thick. If the index of refraction for this glass is 1.52, what is the critical angle of ocean water? How far will a ray of light travel in the glass if it undergoes one internal reflection at the critical angle?

4/28/2016 Snell's Law Prob A

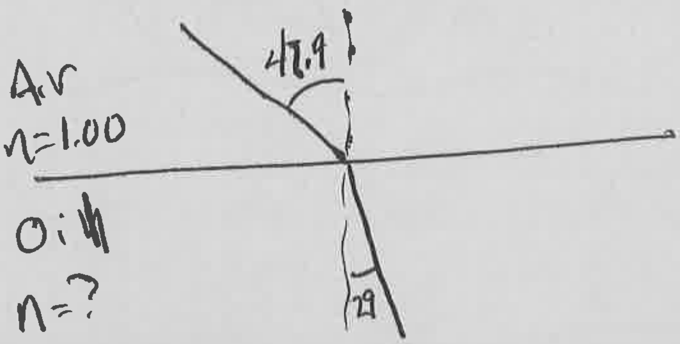
(1)



$n_1 = 1.00$
$\theta_1 = 72.0°$
$n_2 = ?$
$\theta_2 = 34°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$1.0 \sin(72) = n_2 \sin(34)$
$0.951 = n_2 (0.5592)$
$n_2 = 1.7007$

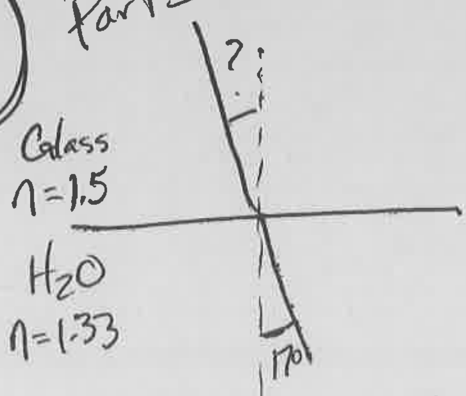
(2)



$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$1.00 \sin(47.9) = n_2 \sin(29)$
$n_2 = 1.53$

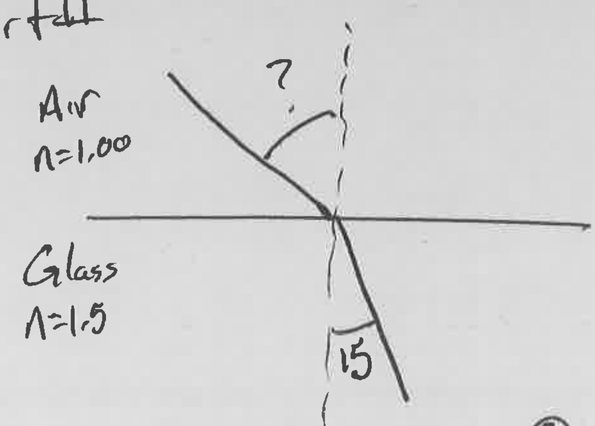
(3)

Part I



$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$1.5 \sin\theta_1 = 1.33 \sin(17)$
$\sin\theta_1 = 0.259$
$\theta = \sin^{-1}(0.259) = 15.0°$

Part II



$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$1.0 \sin\theta_1 = 1.5 \sin(15)$
$\theta_1 = 22.8°$

Prob B Concave Mirrors

(1) $d_o = 370\text{km}$ (stated in problem)

→ Find image distance (d_i)

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{250} = \frac{1}{370} + \frac{1}{d_i}$$

$$d_i = 770.8\text{ km}$$

→ determine magnification

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$m = -\frac{770.8}{370}$$

$$m = -2.08$$

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(5)

$h_o = 32\text{m}$

$f = 120\text{m}$

$d_o = 180\text{m}$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{120} = \frac{1}{180} + \frac{1}{d_i}$$

$$d_i = 360\text{m}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{32} = -\frac{360}{180}$$

$$h_i = -64\text{m}$$

Real → $+d_i$

Inverted → $-h_i$

Enlarged

↳ $h_i > h_o$

Prob C Convex mirrors

(1)
$f = -3.2\times10^3 km$
$d_o = 3.84\times10^5 km$
$h_o = 3475 km$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{-3.2\times10^3} = \frac{1}{3.84\times10^5} + \frac{1}{d_i}$$

$$d_i = -3173.6 km$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{3475} = -\frac{-3173.6}{3.84\times10^5} \longrightarrow h_i = 28.72 m$$

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(2)
$d_o = 553 m$
$R = 120 m$
$f = -60 m$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{-60} = \frac{1}{553} + \frac{1}{d_i}$$

$$d_i = -54.1 m$$

$$m = -\frac{d_i}{d_o}$$

$$m = -\frac{-54.1}{553}$$

$$m = 0.0979$$